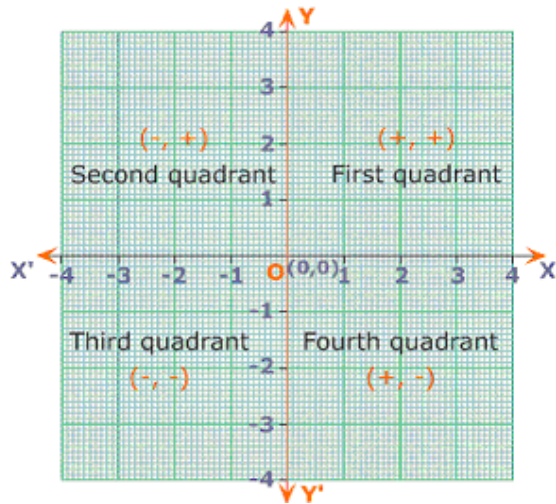


Co-ordinate Geometry

Important Terms and Concepts



“If a pair of perpendicular lines XOX' and YOY' intersect at O , then these lines are called the **co-ordinate axes**”. The axes divide the plane into four quadrants.

The plane containing the axes is called the **Cartesian plane**.

The lines XOX' and YOY' are usually drawn horizontally and vertically as shown in the figure, and are known as x -axis and y -axis respectively.

O , the point of intersection of the axes is called the **origin**.

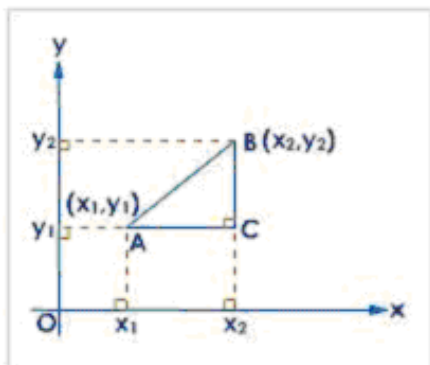
Values of x are measured from O along the x -axis and are called **abscissae**. Along OX , x has positive values while OX' , x has negative values.

Similarly, the values of y are measured from O along the y -axis and are called **ordinate**. Along OY , y has positive values while OY' , y has negative values.

The ordered pair containing the abscissa and the ordinate of a point is called the **coordinates of the point**.

Distance Formula:

To find the distance two points A (x_1, y_1) and B(x_2, y_2)



From the figure

$$AC = x_2 - x_1$$

$$BC = y_2 - y_1$$

∴ In $\triangle ABC$,

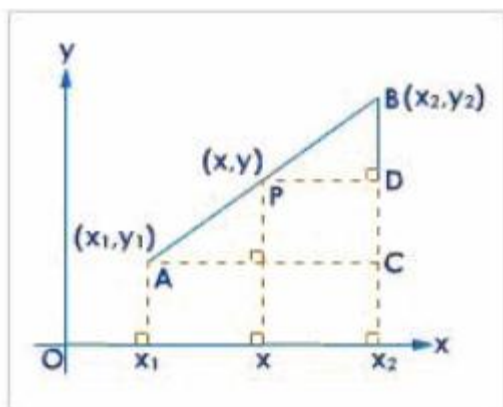
$$AB^2 = AC^2 + BC^2 \text{ (By Pythagoras Theorem)}$$

$$= (x_2 - x_1)^2 + (y_2 - y_1)^2$$

$$\therefore AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Section Formula:

To find the coordinates of a point which divides the line segment joining two given points in a given ratio (internally)



Let P(x, y) divide the join of A(x_1, y_1) and B(x_2, y_2) in the ratio $m : n$

$$\therefore AC = x - x_1, PD = x_2 - x$$

$$\therefore \text{From similarity property } \frac{AC}{PD} = \frac{m}{n} \Rightarrow \frac{x - x_1}{x_2 - x} = \frac{m}{n}$$

Now, Make x the subject of the formula, $nx - nx_1 = mx_2 - mx$

$$mx + nx = mx_2 + nx_1$$

$$x(m + n) = mx_2 + nx_1$$

$$\therefore x = \frac{mx_2 + nx_1}{m+n}$$

Similarly, we can show that

$$y = \frac{my_2 + ny_1}{m+n}$$

Thus coordinate of P are $\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$

Mid-Point Formula:

If P is the mid-point of AB, then $m=n$,

\therefore The ratio becomes 1:1

$$\therefore x = \frac{mx_2 + nx_1}{m+n} = \frac{x_2 + x_1}{1+1} = \frac{x_1 + x_2}{2}$$

Similarly, we get $y = \frac{y_1 + y_2}{2}$

Thus coordinates of point are $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

Note:

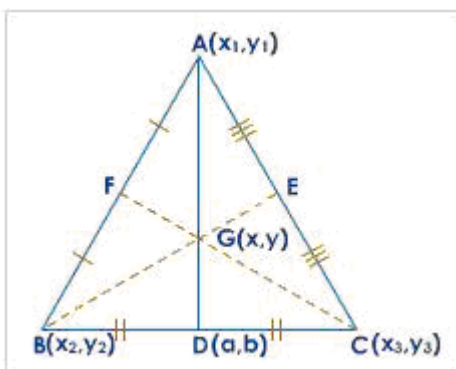
When the point P divides the line joining AB in the ratio $m:n$ externally then

$$P = \left(\frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n}\right)$$

Centroid of a Triangle

Centroid is the point of intersection of three medians. It is the point of intersection of a median

AG: GD is 2: 1



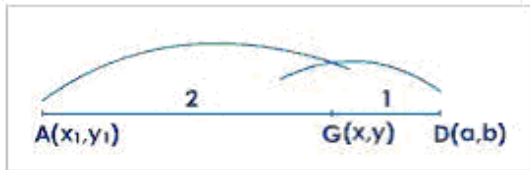
To find the coordinates of the centroid of a triangle.

Let the coordinates of the vertices of $\triangle ABC$, be $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$.

Let $G(x, y)$ be the centroid of the $\triangle ABC$.

By applying the mid-point formula

$$a = \frac{x_1 + x_2}{2} \text{ and } b = \frac{y_1 + y_2}{2}$$



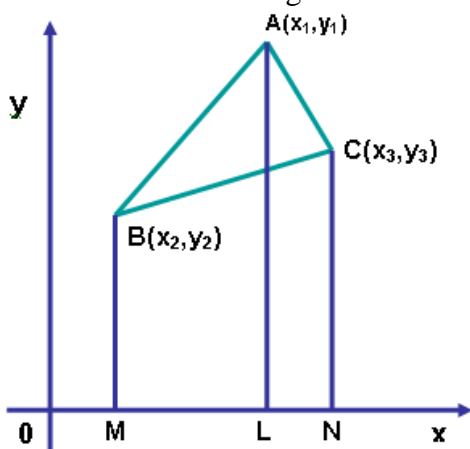
Again, by applying the section formula

$$\begin{aligned} x &= \frac{mx_2 + nx_1}{m+n} \\ &= \frac{2(a) + 1(x_1)}{2+1} \\ &= \frac{2\left(\frac{x_2 + x_3}{2}\right) + x_1}{3} \\ &= \frac{x_1 + x_2 + x_3}{3} \\ &= \frac{2(b) + 1(y_1)}{2+1} \\ &= \frac{2\left(\frac{y_2 + y_3}{2}\right) + y_1}{3} \\ &= \frac{y_1 + y_2 + y_3}{3} \end{aligned}$$

\therefore Coordinates of the centroid are $\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$

Area of the Triangle

To find the area of triangle whose vertices are (x_1, y_1) , (x_2, y_2) and (x_3, y_3) .



Let $A(x_1, y_1), B(x_2, y_2)$ and $C(x_3, y_3)$ be the vertices of a triangle ABC

Area of $\triangle ABC$ = Area of Trapezium ABML

+ Area of Trapezium ALNC

– Area of Trapezium BMNC

$$= \frac{1}{2} ML (MB + LA) + \frac{1}{2} (LN) (LA + NC)$$

$$- \frac{1}{2} (MN) (MB + NC)$$

$$= \frac{1}{2} (x_1 - x_2)(y_2 + y_1) + \frac{1}{2} (x_3 - x_1) (y_1 + y_3)$$

$$- \frac{1}{2} (x_3 - x_2) (y_2 + y_3)$$

$$= \frac{1}{2} \{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}$$

Arrow Method:

It is to obtain the formula for the area of the triangle

$$\frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$= \frac{1}{2} [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

Note:

1. If the points A, B and C we take in the anticlockwise direction, then the area will be positive. If the points we take in clockwise direction the area will be negative.

So we always take the absolute value of the area calculated.

Area of triangle

$$= \frac{1}{2} [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

2. If the area of a triangle is zero, then the three points are collinear.